



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

328. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

If $x^2 + xy + y^2 = 3a^2$, find the maximum value of $bx + cy$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

$$x^2 + xy + y^2 = 3a^2, \quad bx + cy = \text{maximum.}$$

$$\therefore \frac{dx}{dy} = -\frac{x+2y}{2x+y} = -\frac{c}{b}, \text{ or } y = \frac{(2c-b)x}{2b-c}.$$

$$\therefore x^2 = \frac{a^2(2b-c)^2}{b^2 - bc + c^2}; \text{ and } x = \pm \frac{a(2b-c)}{\sqrt{b^2 - bc + c^2}}, \quad y = \pm \frac{a(2c-b)}{\sqrt{b^2 - bc + c^2}}.$$

$$\therefore bx + cy = 2a\sqrt{b^2 - bc + c^2} = 2a\sqrt{\frac{b^3 + c^3}{b+c}} \text{ is a maximum.}$$

Also solved by F. L. Griffin and S. G. Barton.

329. Proposed by C. N. SCHMALL, 604 East 5th Street, New York City.

Between the quantities a and b there are inserted n arithmetical and n harmonical means, and a series of n terms is formed by dividing each arithmetical by the corresponding harmonical mean. Show that the sum of the series is, $n \left[1 + \frac{n+2}{n+1} \cdot \frac{(a-b)^2}{6ab} \right]$.

Solution by HOWARD C. FEEMSTER, A. B., Professor of Mathematics, York College, York, Neb., and S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

The n arithmetic means between a and b are:

$$(1) \frac{b+na}{n+1}, \quad \frac{2b+(n-1)a}{n+1}, \quad \dots, \quad \frac{rb+(n-r+1)a}{n+1}, \quad \dots$$

The n harmonic means between a and b are:

$$(2) \frac{ab(n+1)}{nb+a}, \quad \frac{ab(n+1)}{(n-1)b+2a}, \quad \dots, \quad \frac{ab(n+1)}{(n-r+1)b+(r-1)a}, \quad \dots$$

Dividing the terms of (1) by the corresponding terms of (2),

$$\frac{(b+na)(a+nb)}{ab(n+1)^2}, \quad \frac{[2b+(n-1)a][(n-1)b+2a]}{ab(n+1)^2}, \quad \dots,$$

$$\frac{[rb + (n-r+1)a][(n-r+1)b + 2a]}{ab(n+1)^2}, \dots, \text{the series to be summed.}$$

$$\text{Hence } S = \frac{ab(n^2+1) + n(a^2+b^2)}{ab(n+1)^2} + \frac{ab[(n-1)^2+r^2] + 2(n-1)(a^2+b^2)}{ab(n+1)^2} + \dots$$

$$+ \frac{ab[(n-r+1)^2+r^2] + r(n-r+1)(a^2+b^2)}{ab(n+1)^2} + \dots$$

$$= \frac{1}{ab(n+1)^2} \{ ab[n^2 + (n-1)^2 + \dots + 1^2] + ab[1^2 + 2^2 + \dots + n^2] \}$$

$$= \frac{(a^2+b^2)[n+2(n-1)+3(n-2)+\dots+r(n-r+1)+\dots+n]}{ab(n+1)^2}$$

$$= \frac{1}{ab(n+1)^2} \left[2ab \frac{n(n+1)(2n+1)}{6} + (a^2+b^2) \left(\frac{n(n^2+1)}{2} - \frac{n(n^2-1)}{3} \right) \right]$$

$$= \frac{1}{ab(n+1)^2} \left[2ab \frac{n(n+1)(2n+1)}{6} + (a^2+b^2) \left(\frac{n(n+1)(n+2)}{6} \right) \right]$$

$$= \frac{n}{ab(n+1)^2} \left[\frac{6ab(n+1)^2}{6} - \frac{n^2+3n+2}{6} \cdot 2ab + \frac{(n+1)(n+2)}{6} (a^2+b^2) \right]$$

$$= n \left[1 + \frac{a^2-2ab+b^2}{6ab(n+1)^2} \cdot (n+1)(n+2) \right] = n \left[1 + \frac{n+2}{n+1} \cdot \frac{(a-b)^2}{6ab} \right].$$

Solved similarly by G. B. M. Zerr, and J. Scheffer.

GEOMETRY.

352. Proposed by G. I. HOPKINS, Professor of Astronomy, High School, Manchester, N. H.

Required, to construct the triangle, having given the base, vertical angle and sum of the altitude and the two remaining sides.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $AB=a$ be the given base; ACB the given vertical angle; p —to the sum of the altitude and the remaining sides. On AB describe the seg—